Algebraisk lösning av trigonometriska ekvationer

– Via grafernas egenskaper

ex: $\sin(2x) = \frac{1}{2}$

ex: $\cos(3x) = \frac{\sqrt{3}}{2}$

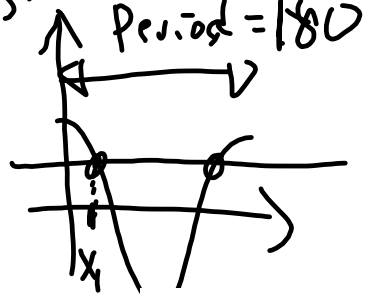
Exakta värden

Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radianer)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Ex: Lös ekv:

$$\cos(2x) = \frac{1}{2}$$

Skiss:



$\frac{360}{2} = 180°$

x_1 fås via FB:s tabell

Exakta värden

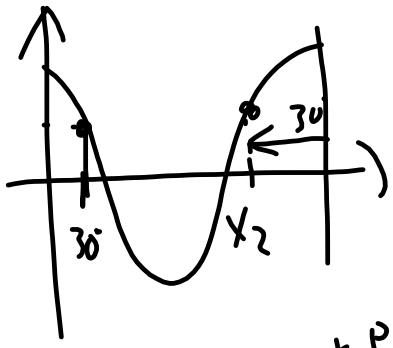
Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radianer)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

$\cos(2x) = \frac{1}{2}$

$2x_1 = 60°$

$x_1 = 30°$

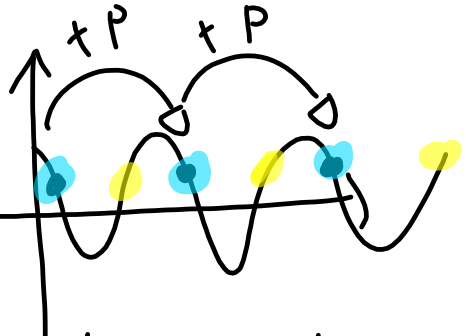
x_2 fås via symmetri:



$x_2 = P - x_1$

$= 180 - 30$

$= 150°$



Alla lösningar:

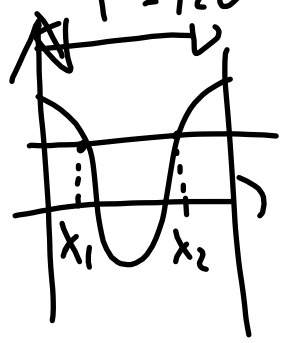
x_1: $x = 30° + 180° \cdot n$

x_2: $x = 150° + 180° \cdot n$

där n är ett heltal

Ex: $\cos(3x) = \frac{1}{\sqrt{2}}$

$P = 120^\circ$



$P = \frac{360}{3} = 120^\circ$

Exakta värden

$3x_1 = 45^\circ$

$x_1 = 15^\circ$

Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radianer)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

$x_1 = 15^\circ$ $\quad x_2 = P - x_1 = 120 - 15 = 105^\circ$

Alla lösningar: x_1: $x = 15^\circ + n \cdot 120^\circ$

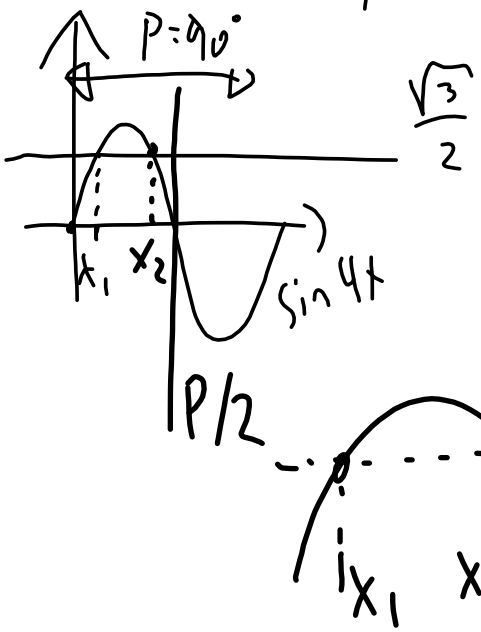
x_2: $x = 105^\circ + n \cdot 120^\circ$

Ex: Lös ekvationen

$\sin(4x) = \frac{\sqrt{3}}{2}$

Exakta värden

Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radianer)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0



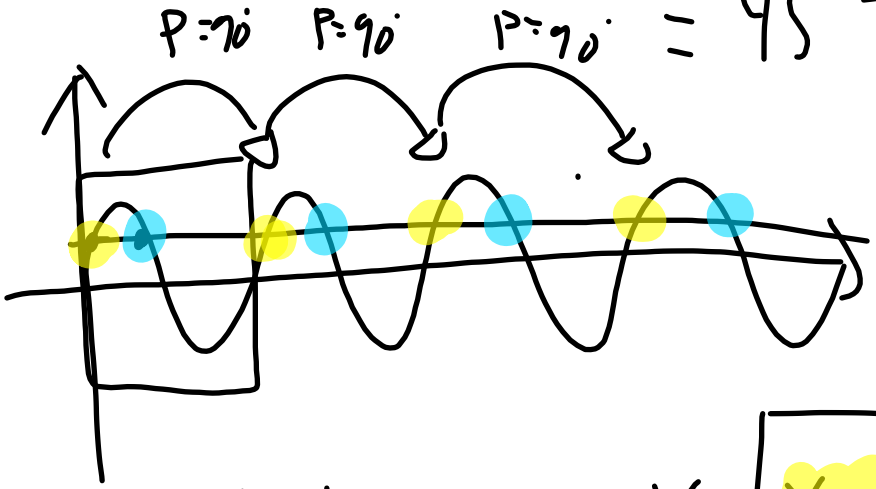
X_1 fås via FB:s tabell

$4 \cdot X_1 = 60° \Rightarrow X_1 = 15°$

X_2 fås via symmetri

$X_2 = P/2 - X_1$

$= 45° - 15° = 30°$



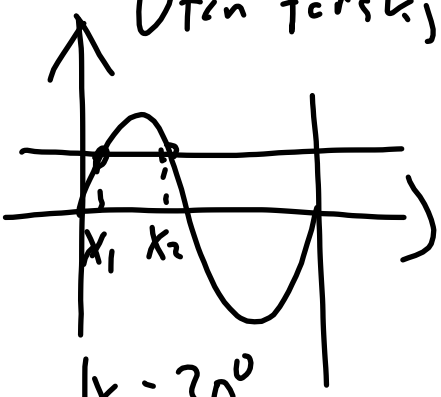
Alla lösningar: X_1: $x = 15° + 90° \cdot n$

X_2: $x = 30° + 90° \cdot n$

Ex: Lös ekv: $\sin(1x - 45°) = \frac{1}{2}$

$P = 360°$

Utan förskjutning



Exakta värden

Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
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$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

$1x_1 = 30°$

$x_2 = P/2 - x_1 = 180 - 30 = 150°$

Med förskjutning: "$-45°$" $\Rightarrow$ 45° åt höger

"x_1" $= 30° + 45° = 75°$

"x_2" $= 150° + 45° = 195°$

Alla lösningar: $x = 75° + n \cdot 360°$

$x = 195° + n \cdot 360°$

$$\sin(1x - 45^\circ) = \frac{1}{2}$$

$1x - 45^\circ = 30^\circ + n \cdot 360^\circ$

$X = 75^\circ + n \cdot 360^\circ$

$1x - 45^\circ = 150^\circ + n \cdot 360^\circ$

$X = 195^\circ + n \cdot 360^\circ$